Year 12 Examination, 2018

Question/Answer Booklet

MATHEMATICS SPECIALIST

Section One: Calculator-free

Student Name/Number: _____

Teacher Name: _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor: This Question/Answer Booklet
Formula Sheet

To be provided by the candidate:
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

See next page

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	54	35
Section Two: Calculator-assumed	13	13	100	100	65
					100

Instructions to candidates

1. The rules for the conduct of School exams are detailed in the _____*School/College assessment policy*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section One: Calculator-free **35% (54 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Suggested working time: 50 minutes.

Question 1 **(5 marks)**

Calculate all four solutions of the equation

$$z^4 = 1 - i\sqrt{3}$$

leaving your answers expressed in polar form with arguments in the range $(-\pi, \pi]$.

See next page

Question 2 **(12 marks)**

(a) Use double-angle formulae to evaluate

$$\int_{\pi/6}^{\pi/4} \frac{dx}{1+\cos 2x} \ .$$ (4 marks)

(b) Use the substitution $u = \sqrt{x}$ to determine $\int_{4}^{9} \frac{dx}{x+\sqrt{x}}$. (4 marks)

See next page

(c) Determine the value of the integer n if

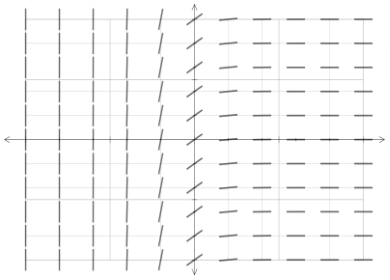
$$\int_0^{\frac{\pi}{2}} \sin x \cos^n x \, dx = \frac{1}{2018} \ .$$ (4 marks)

See next page

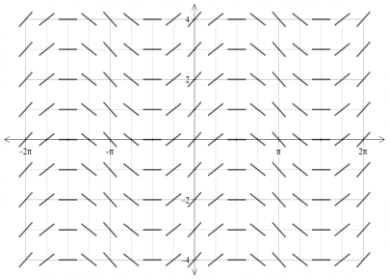
Question 3 **(9 marks)**

(a) Match the following three slope fields with their differential equations. (2 marks)

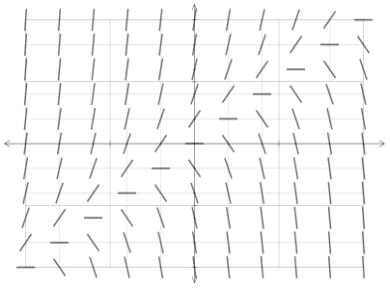
(A)



(B)



(C)



I. $\frac{dy}{dx} = e^{-x}$ II. $\frac{dy}{dx} = y - x$ III. $\frac{dy}{dx} = \cos x$

Slope field A corresponds to differential equation ……

Slope field B corresponds to differential equation ……

Slope field C corresponds to differential equation ……

See next page

(b) Consider the differential equation $\frac{dy}{dx} = x(y-2)^2$.

(i) Let $y = f(x)$ be the particular solution of this differential equation with the initial condition $f(0) = -2$.

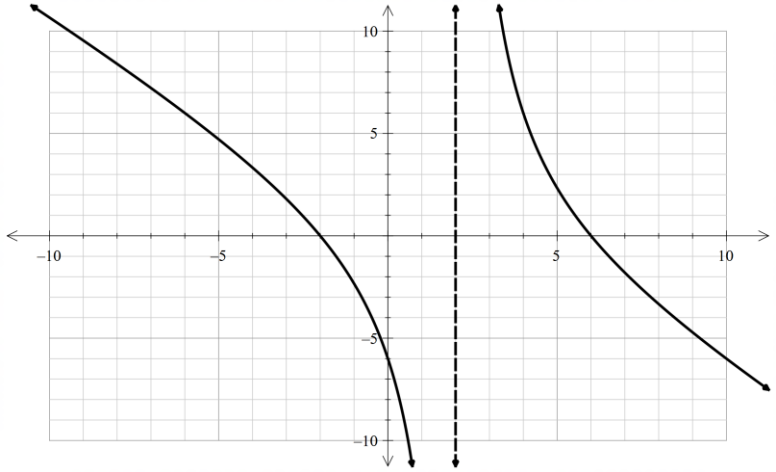
Does f have a relative maximum, a relative minimum, or neither at $x = 0$?
Justify your answer. (3 marks)

(ii) Determine the particular solution to the given differential equation with the initial condition $f(0) = -2$. (4 marks)

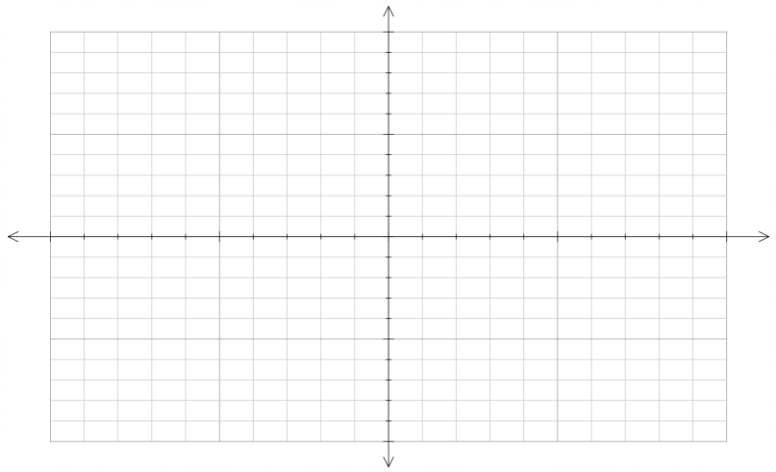
See next page

Question 4 **(6 marks)**

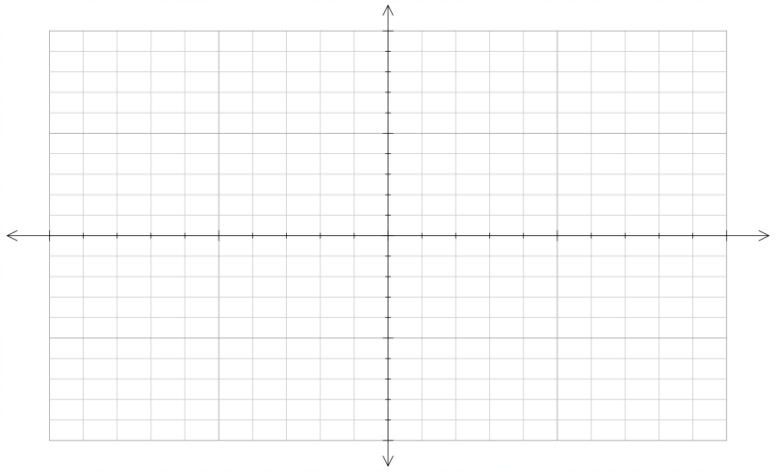
The graph of $y = f(x)$ is shown below.



(a) Sketch the graph of $g(x) = f(|x|)$. (3 marks)



(b) Sketch the graph of $h(x) = \dfrac{1}{f(x)}$. (3 marks)



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Question 5 **(8 marks)**

The Cartesian equation of a sphere S is

$$x^2 + y^2 + z^2 - 8x + 12y - 24z + 171 = 0 .$$

(a) Determine the radius of S and the co-ordinates of its centre. (2 marks)

(b) Determine the co-ordinates of the point A on the sphere that is closest to the origin.

(3 marks)

(c) Determine the equation of the plane P which is tangential to S at the point A. (3 marks)

See next page

Question 6 **(9 marks)**

Ten 90% confidence intervals for the mean μ will be constructed from ten independent random samples of the same size taken from the same population.

State whether the following statements about these confidence intervals are true or false, and give a brief justification for your answers.

(a) Any one of the confidence intervals will contain 90% of the population. (2 marks)

(b) The probability that any one of the confidence intervals will contain μ is 0.9. (2 marks)

(c) Exactly 9 of the 10 confidence intervals will contain μ. (2 marks)

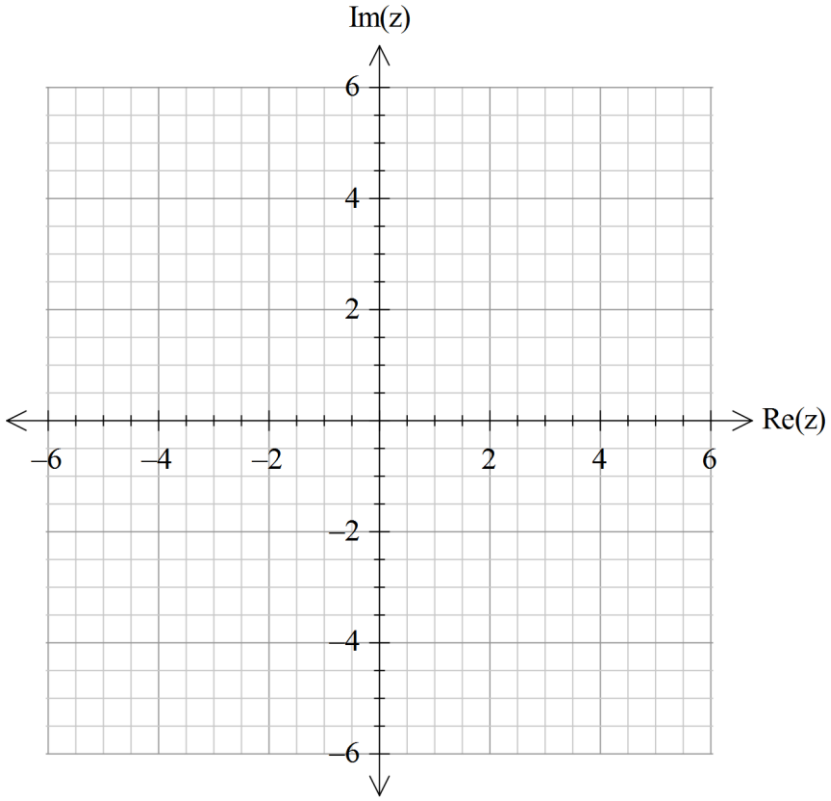
(d) The probability that exactly 9 of the 10 confidence intervals will contain μ is greater than the probability that all of the confidence intervals will contain μ. (3 marks)

See next page

Question 7 **(5 marks)**

Sketch on the axes below the set of all complex numbers z which satisfy

$$|z+2| \leq 2 \quad \text{and} \quad -\frac{\pi}{2} \leq \arg(z+2) \leq \frac{\pi}{4}.$$



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Acknowledgements

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